## 

# On Efficient Indexing for Distance Queries between Arbitrary Points on Terrain Surface 

Bo Huang (SUSTech), Victor Junqiu Wei (PolyU), Raymond Chi-Wing Wong (HKUST), and Bo Tang (SUSTech)

## Overview

- The distance computation on the terrain surface is a fundamental and important problem that is widely applied in geographical information systems and 3D modeling.
- We propose an indexing structure, namely Efficient Arbitrary Point-to-Arbitrary Point Distance Oracle (EAR-Oracle), with theoretical guarantee on the accuracy, oracle building time, oracle size and query time. Our experiments demonstrate that our oracle outperforms the state-of-the-art algorithms by orders of magnitude.



## Preliminary

- Terrain surface: Consists of Vertices, Edges and Faces in 3D.
- Geodesic distance: Shortest distance on the terrain surface.
- Arbitrary point-to-arbitrary point (A2A) queries: Geodesic distance queries between arbitrary surface points.


## Application

- Geographic Information Systems (GIS):
- Compute travel cost;
- Study animal travel patterns.
- Spatial Data Mining:
- Check spatial co-location patterns;
- Clustering objects on terrain surfaces.
- Scientific 3D Modeling:
- Analyze key features.



## Research Problem

- Given two arbitrary surface points $s$ and $t$, find the approximate geodesic distance between $s$ and $t$ with theoretical guarantee.


## Existing Study

- On-the-fly algorithms:
- Fixed Scheme (FS) [Algorithmica' 2001]
- Unfixed Scheme (US) [J. ACM' 2005]
- K-Algorithm (K-Algo) [VLDB' 2015]
*Deficiency: Queries are processed online (large query latency).
- Index-based algorithms:
- Steiner-Point Oracle (SP-Oracle) [ESA' 2011]
- Space-Efficient Oracle (SE-Oracle) [SIGMOD' 2017]
* Deficiency: Index too many auxiliary points (large index cost).


## Contribution

- We propose an index-based algorithm for A2A queries, Efficient Arbitrary point-to-arbitrary point distance oracle (EAR-Oracle):
- No assumptions on query points;
- Outperforms state-of-the-art index-based algorithm by 2 orders of magnitude in terms of building time and space consumption;
- Outperforms the fastest on-the-fly algorithm by more than 1 order of magnitude in terms of query time;
- We provide thorough theoretical analysis:
- Building time, space consumption, query time and relative error.
- We conduct extensive experimental studies:
- On several real datasets with different scales;
- On factors influencing the performance of EAR-Oracle.


## Theoretical Analysis

- Let $N$ be the number of terrain faces and $\epsilon$ be the user-defined error bound:
- Building time: linearithmic to $N$;

A subset of terrain vertices whose

- Space consumption: linear to N;
cardinality is much less than $N$.
- Query time: linearithmic to the amount of highway nodes;
- Distance relative error: close to $\epsilon$ in practice.

EAR-Oracle Flow Chart


## Proposed Method: EAR-Oracle

- Pre-processing Phase:
- Construct base graph $\left(G_{B}\right)$ for distance metric approximation:
- Place $m$ Steiner points uniformly on each angle bisector.
- Partition terrain surface into boxes on $x-y$ plane:
- When the query points are close, they have spatial locality;
- When the query points are distant, their geodesic path will go through boundaries of some boxes.
- Select highway nodes based on terrain vertices:
- A subset of the terrain vertices near box boundaries.
- Construct highway network based on highway nodes:

Generate edges between highway nodes according to geometric property (avoid all-pair distances computation).

- Build distance map based on highway nodes and Steiner points:

For each box, index the distance between each highway node on its boundaries and Steiner points on the faces inside it.

(a) Highway Nodes and Steiner Points

(b) Highway Edges and Distance Map

- Query Phase:
- Inner-box queries (Query points in the same box):
- Adopt Dijkstra's algorithm on base graph $G_{B}$.
- Inter-box queries (Query points in different boxes):
- Construct a query graph $G_{Q}$ by adding edges (from the distance map) to the highway network;
Perform Dijkstra's algorithm on query graph $G_{Q}$.

(a) Shortest Distance Queries

(b) Inner-Box Query

(c) Inter-Box Query

Experimental Study

- We test both on-the-fly and index-based algorithms.
- EAR-Oracle has the best overall performance.


| Dataset | No. of Faces | Region Covered |
| :---: | :---: | :---: |
| HorseMountain (HM) | 1,488 | 15 km ${ }^{2}$ |
| BigMountain (BM) | 2,772 | $29 \mathrm{~km}^{2}$ |
| HeadLightMountain (HL) | 4,771 | $49 \mathrm{~km}^{2}$ |
| RobinsonMountain (RM) | 7,200 | 71 km ${ }^{2}$ |
| GunnisonForest (GF) | 199,998 | $10,038 \mathrm{~km}^{2}$ |
| LaramieMountain (LM) | 199,996 | $12,400 \mathrm{~km}^{2}$ |
| BearHead (BH) | 292,914 | $140 \mathrm{~km}^{2}$ |
| EaglePeak (EP) | 325,713 | $150 \mathrm{~km}^{2}$ |

